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the mortality of London, as derived from the parish registers : which, when thus incorporated with tables formed in the country, will be freed from the objections that have been made to the observations of burials in great cities only.

The Carlisle Table agrees in the earlier parts pretty nearly with the observations of Mr. Morgan on the experience of the Equitable Office from 1768 to 1810, as it appears from Mr. Milne's comparison, as well as from the reduction and interpolation of those observations, published by Mr. Gompertz in the *Philosophical Transactions* for 1825 ; but for correcting the later portions of the Carlisle Table, it may be allowable to employ a subsequent register of the experience of the Equitable Office, so far as it is possible to make any inferences from it with safety.

The numbers of deaths occurring in twenty years, as recorded by Mr. Morgan, might have been made the foundation of a very valuable determination of the mortality occurring in a certain class of persons, if the number of the Equitable Society had become stationary before the commencement of the record ; but in order to deduce from it a just estimate of the value of life, it would then be necessary to alter the numbers of deaths at each age, in the inverse proportion of the numbers of the living compared—that is to say, not simply of the sums of the persons admitted under that age, but of the numbers of persons born whom they represent ; since, in comparing the joint mortalities of any two lists of persons, we must obviously add together the deaths belonging, not to a given number of persons of various ages, but of a number proportionate to the survivors at the respective ages out of a given number of births : so that in this manner the apparent mortality in the earlier portions of the register would require to be augmented, not only on account of the smaller number of persons who have actually contributed to furnish it, but also on account of the greater proportion that these persons bear to the corresponding number at birth, when compared with the survivors at more advanced ages, who represent a population still more exceeding their own numbers. On the other hand, since the register in question relates only to a limited number of years, immediately following a very rapid increase of the Society, it is evident that the deaths must have occurred at earlier ages than if it had been continued until all the lives had dropped.

Of these three modifications, it may be sufficiently accurate for the present purpose to omit the two latter, as nearly counterbalancing each other, and to augment the earlier numbers in the proportion only of the members of the Society to whom they must necessarily

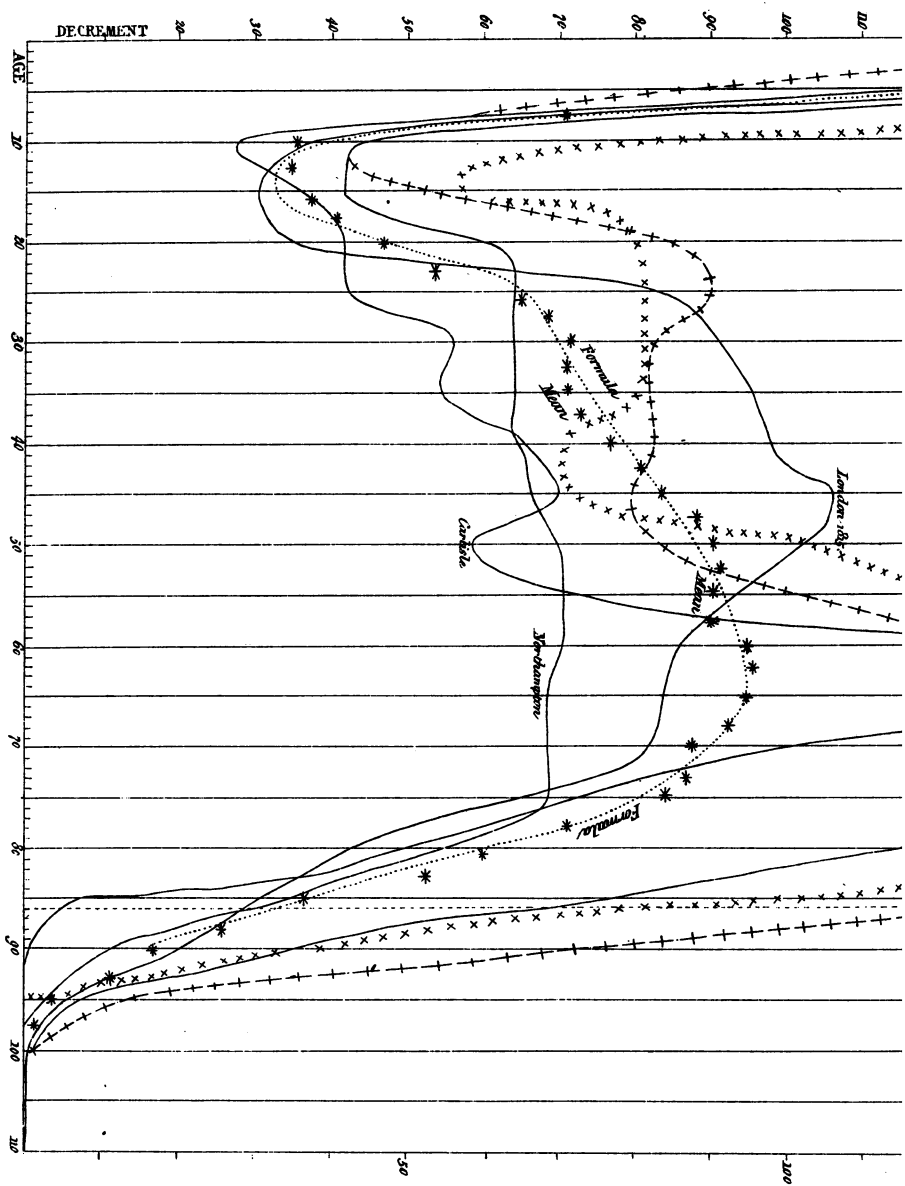
have belonged, supposing that the admissions had taken place about the same ages at all periods; assuming also the number of survivors at 45 to be in the same proportion to the births as in the Carlisle Table. We may then proceed to take a mean between the mortality thus obtained, with proper interpolations, and the observations at Carlisle, as the second of the three principal bases to be afterwards incorporated with the mortality of Northampton and of London. Further than this, it is impossible to place any great reliance on Mr. Morgan's document, which makes the annual deaths, in "a population exceeding 150,000," not quite 1 in 1,500.

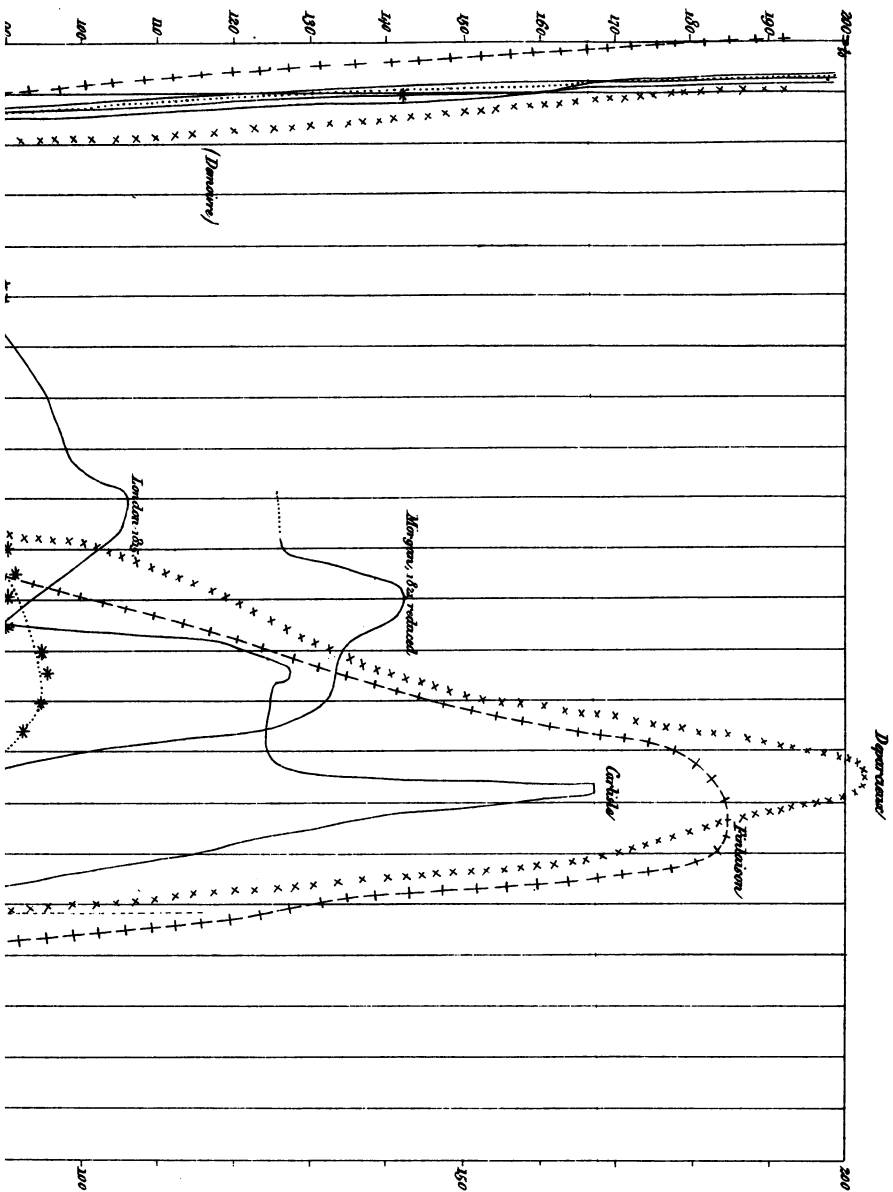
Of the mortality of London, taken for the ten years from 1811 to 1820, it may be observed, that its results bear the internal evidence of greater apparent correctness than either of the other bases, exhibiting a curve less irregular in its flexures, and generally intermediate between the others: it has also the advantage of exhibiting the duration of life as prolonged by the general introduction of vaccination: and when thus incorporated with the registers of two places in the country, each reduced to an equal supposed population, it must probably be sufficiently corrected for the errors that may be attributed to the effect of an afflux of settlers at an early age. The mean obtained in this manner might be employed at once as a standard table, without much inconvenience; but it still exhibits some minute but obvious irregularities, as an inspection of the line of stars in the diagram will show, principally perhaps from the want of skill or care with which the interpolations have been made by Dr. Price and others. The most effectual of all interpolations for *harmonizing* the various orders of differences is to obtain a formula which shall extend with sufficient accuracy throughout the whole curve. It may be easily believed that it must be extremely difficult to find such an expression, and that its form must be too complicated to be applied to any practical purpose throughout its extent. I have, however, drawn a curve which comes extremely near to the line of stars, and crosses it in ten or twelve different points, by means of the equation

$$y = 368 + 10x - 11 \cdot (156 + 20x - x^2)^{\frac{3}{2}} + \frac{1}{2.85 + 2.05x^2 + 2\left(\frac{x}{10}\right)^6} \\ - 5 \cdot 5\left(\frac{x}{50}\right)^{10} + \frac{5 \cdot 5^2}{4000}\left(\frac{x}{50}\right)^{20} - 5500\left(\frac{x}{100}\right)^{40} :$$

$y$  being the number of deaths among 100,000 persons, in the year that completes the age  $x$ .

The terms of this formula have some remarkable relations to the different periods of life. Halley's first approximation was





$y=1,000$ , throughout life. De Moivre's arithmetical hypothesis was  $y = \frac{100,000}{86} = 1163$ : but of the present formula the principal foundation, as extending to the whole of life, is  $y=368+10x$ . In infancy, the term containing the reciprocal of the powers of  $x$  has a preponderating value; in youth, the term  $-(156+20x-x^2)^{\frac{3}{2}}$ , which diminishes the mortality, ends somewhat abruptly at 25, and would be incapable of being employed with safety in algebraical calculations, from its having a negative as well as a positive value. Old age is expressed almost exclusively by the high powers at the end of the formula, which terminate the series with great and increasing rapidity. It is obvious that, for many purposes of calculation, the terms belonging to youth and to old age might be neglected without inconvenience, and that, for the middle portion of life, the terms  $368+10x$  alone, with some little modification, might be employed as sufficiently correct; or, certainly, as much nearer to the truth than either the arithmetical or geometrical hypothesis of De Moivre. The relations of the different parts of the formula will be best appreciated from their development in the following tables.

*Decrements of Mortality, computed from the Formula.*

| Age<br>( $x-1$ ) | $368+10x$ | $-11(156+20x-x^2)^{\frac{3}{2}}$ | $+ \frac{1}{2.85+2.05x^2+2\left(\frac{x}{10}\right)^6}$ | Decrement. |
|------------------|-----------|----------------------------------|---|------------|
| 0                | 378       | -255                             | +20,408   | 20,531     |
| 1                | 388       | 241                              | 9,009   | 9,106      |
| 2                | 398       | 313                              | 4,695   | 4,780      |
| 3                | 408       | 359                              | 2,805   | 2,854      |
| 4                | 418       | 386                              | 1,848   | 1,880      |
| 5                | 428       | 409                              | 1,322   | 1,341      |
| 6                | 438       | 427                              | 968   | 979        |
| 7                | 448       | 440                              | 746   | 752        |
| 8                | 458       | 447                              | 592   | 603        |
| 9                | 468       | 451                              | 477   | 494        |
| 10               | 478       | 447                              | 392   | 423        |
| 11               | 488       | 440                              | 329   | 377        |
| 12               | 498       | 427                              | 278   | 349        |
| 13               | 508       | 409                              | 238   | 337        |
| 14               | 518       | 386                              | 205   | 337        |
| 15               | 528       | 359                              | 178   | 347        |
| 16               | 538       | 313                              | 156   | 381        |
| 17               | 548       | 291                              | 136   | 393        |
| 18               | 558       | 255                              | 119   | 422        |
| 19               | 568       | 214                              | 104   | 458        |
| 20               | 578       | 174                              | 93  | 497        |
| 21               | 588       | 130                              | 82  | 540        |
| 22               | 598       | 89                               | 72  | 581        |
| 23               | 608       | 51                               | 64  | 621        |
| 24               | 618       | 19                               | 57  | 656        |
| 25               | 628       | 0                                | 50  | 678        |

*Decrement of Mortality, computed from the Formula—(continued).*

| Age<br>( $x-1$ ) | 368<br>+ 10 $x$ | $+ \frac{1}{2 \cdot 85 + 2 \cdot 05x^2 + 2 \left(\frac{x}{10}\right)^6}$ | $- 5 \cdot 5 \left(\frac{x}{50}\right)^{10}$ | $+ \cdot 001 \left(\frac{5 \cdot 5 \left(\frac{x}{50}\right)^{10}}{2}\right)^2$ | Decre-<br>ment. |
|------------------|-----------------|--|--|---|-----------------|
| 26               | 638             | 44   | ..   | ..  | 682             |
| 27               | 648             | 39   | ..   | ..  | 687             |
| 28               | 658             | 34   | ..   | ..  | 692             |
| 29               | 668             | 30   | ..   | ..  | 698             |
| 30               | 678             | 27   | ..   | ..  | 705             |
| 31               | 688             | 24   | ..   | ..  | 712             |
| 32               | 698             | 21   | ..   | ..  | 719             |
| 33               | 708             | 18   | ..   | ..  | 726             |
| 34               | 718             | 16   | ..   | ..  | 734             |
| 35               | 728             | 14   | ..   | ..  | 742             |
| 36               | 738             | 13   | ..   | ..  | 751             |
| 37               | 748             | 11   | — 0  | ..  | 759             |
| 38               | 758             | 10   | ·4   | ..  | 768             |
| 39               | 768             | 9  | ·6   | ..  | 776             |
| 40               | 778             | 8  | ·7   | ..  | 785             |
| 41               | 788             | 8  | ·9   | ..  | 795             |
| 42               | 798             | 7  | 1·2  | ..  | 804             |
| 43               | 808             | 6  | 1·5  | ..  | 813             |
| 44               | 818             | 5  | 1·9  | ..  | 821             |
| 45               | 828             | 5  | 2·3  | ..  | 831             |
| 46               | 838             | 4  | 3·0  | ..  | 839             |
| 47               | 848             | 4  | 3·9  | ..  | 848             |
| 48               | 858             | 3  | 4·5  | ..  | 857             |
| 49               | 868             | 3  | 5·5  | ..  | 866             |
| 50               | 878             | 3  | 6·7  | ..  | 874             |
| 51               | 888             | 2  | 8  | ..  | 882             |
| 52               | 898             | 2  | 10   | ..  | 890             |
| 53               | 908             | 2  | 12   | ..  | 898             |
| 54               | 918             | 2  | 14   | ..  | 906             |
| 55               | 928             | —2   | 17   | ..  | 913             |
| 56               | 938             | 1  | 20   | ..  | 917             |
| 57               | 948             | 1  | 24   | ..  | 923             |
| 58               | 958             | 1  | 28   | ..  | 929             |
| 59               | 968             | 1  | 33   | ..  | 934             |
| 60               | 978             | 1  | 39   | + 0   | 938             |
| 61               | 988             | 1  | 46   | 1   | 942             |
| 62               | 998             | 1  | 55   | 1   | 943             |
| 63               | 1,008           | 1  | 64   | 1   | 944             |
| 64               | 1,018           | 1  | 75   | 1   | 943             |
| 65               | 1,028           | ..   | —88  | 2   | 942             |
| 66               | 1,038           | ..   | 102  | 3   | 939             |
| 67               | 1,048           | ..   | 119  | 4   | 933             |
| 68               | 1,058           | ..   | 137  | 5   | 926             |
| 69               | 1,068           | ..   | 159  | 6   | 915             |
| 70               | 1,078           | ..   | 183  | 8   | 903             |
| 71               | 1,088           | ..   | 211  | 11  | 888             |
| 72               | 1,098           | ..   | 242  | 15  | 871             |
| 73               | 1,108           | ..   | 277  | 19  | 850             |
| 74               | 1,118           | ..   | 317  | 25  | 826             |



*Decrements of Mortality, computed from the Formula—(continued).*

| Age<br>( $x-1$ ) | $368 + 10x$ | $-5.5 \left( \frac{x}{50} \right)^{10}$ | $+ .001 \left( \frac{5.5 \left( \frac{x}{50} \right)^{10}}{2} \right)^2$ | $-5500 \left( \frac{x}{100} \right)^{40} =$ | Decre-<br>ment. |
|------------------|-------------|---|--|---|-----------------|
| 75               | 1,128       | 359                                     | 32   | ..  | 801             |
| 76               | 1,138       | 412                                     | 42   | ..  | 768             |
| 77               | 1,148       | 470                                     | 55   | ..  | 733             |
| 78               | 1,158       | 532                                     | 71   | — 0   | 697             |
| 79               | 1,168       | 604                                     | 91   | 1   | 654             |
| 80               | 1,178       | 684                                     | 117  | 1   | 610             |
| 81               | 1,188       | 772                                     | 145  | 2   | 559             |
| 82               | 1,198       | 872                                     | 190  | 3   | 513             |
| 83               | 1,208       | 984                                     | 242  | 6   | 460             |
| 84               | 1,218       | 1,108                                   | 307  | 9   | 403             |
| 85               | 1,228       | 1,243                                   | 386  | 14  | 357             |
| 86               | 1,238       | 1,399                                   | 490  | 22  | 307             |
| 87               | 1,248       | 1,567                                   | 614  | 37  | 258             |
| 88               | 1,258       | 1,756                                   | 771  | 52  | 215             |
| 89               | 1,268       | 1,963                                   | 963  | 86  | 178             |
| 90               | 1,278       | 2,192                                   | 1,201  | 139   | 148             |
| 91               | 1,288       | 2,444                                   | 1,493  | 212   | 125             |
| 92               | 1,298       | 2,713                                   | 1,849  | 333   | 101             |
| 93               | 1,308       | 3,032                                   | 2,300  | 496   | 80              |
| 94               | 1,318       | 3,371                                   | 2,840  | 734   | 53              |
| 95               | 1,328       | 3,744                                   | 3,504  | 1,041                                       | 27              |
| 96               | 1,338       | 4,150                                   | 4,306  | 1,746                                       | 3               |
| 97               | 1,348       |   |  |   |                 |
| 98               | 1,358       |   |  |   |                 |
| 99               | 1,368       |   |  |   |                 |
| 100)             | 1,378       |   |  |   |                 |

*Mean Standard Table of the Decrements of Life in Great Britain,  
1824.*

| Age. | Decre-<br>ment. | Living. | Age. | Decre-<br>ment. | Living. | Age. | Decre-<br>ment. | Living. | Age. | Decre-<br>ment. | Living. |
|------|-----------------|---------|------|-----------------|---------|------|-----------------|---------|------|-----------------|---------|
| 0    | 20,531          | 100,003 | 15   | 347             | 54,860  | 30   | 705             | 46,527  | 45   | 831             | 35,117  |
| 1    | 9,106           | 79,472  | 16   | 381             | 54,513  | 31   | 712             | 45,822  | 46   | 839             | 34,286  |
| 2    | 4,780           | 70,366  | 17   | 393             | 54,132  | 32   | 719             | 45,110  | 47   | 848             | 33,447  |
| 3    | 2,854           | 65,586  | 18   | 422             | 53,739  | 33   | 726             | 44,391  | 48   | 857             | 32,599  |
| 4    | 1,880           | 62,732  | 19   | 458             | 53,317  | 34   | 734             | 43,665  | 49   | 866             | 31,742  |
| 5    | 1,341           | 60,852  | 20   | 497             | 52,859  | 35   | 742             | 42,931  | 50   | 874             | 30,876  |
| 6    | 979             | 59,511  | 21   | 540             | 52,362  | 36   | 751             | 42,189  | 51   | 882             | 30,002  |
| 7    | 752             | 58,532  | 22   | 581             | 51,822  | 37   | 759             | 41,438  | 52   | 890             | 29,120  |
| 8    | 603             | 57,780  | 23   | 621             | 51,241  | 38   | 768             | 40,679  | 53   | 898             | 28,230  |
| 9    | 494             | 57,177  | 24   | 656             | 50,620  | 39   | 776             | 39,911  | 54   | 906             | 27,332  |
| 10   | 423             | 56,683  | 25   | 678             | 49,964  | 40   | 785             | 39,135  | 55   | 913             | 26,426  |
| 11   | 377             | 56,260  | 26   | 682             | 49,286  | 41   | 795             | 38,350  | 56   | 917             | 25,513  |
| 12   | 349             | 55,883  | 27   | 687             | 48,604  | 42   | 804             | 37,555  | 57   | 923             | 24,596  |
| 13   | 337             | 55,534  | 28   | 692             | 47,917  | 43   | 813             | 36,751  | 58   | 929             | 23,673  |
| 14   | 337             | 55,197  | 29   | 698             | 47,225  | 44   | 821             | 35,938  | 59   | 934             | 22,744  |

Mean Standard Table of the Decrements of Life in Great Britain, 1824—  
(continued).

| Age. | Decre-<br>ment. | Living. | Age. | Decre-<br>ment. | Living. | Age. | Decre-<br>ment. | Living. | Age. | Decre-<br>ment. | Living. |
|------|-----------------|---------|------|-----------------|---------|------|-----------------|---------|------|-----------------|---------|
| 60   | 938             | 21,810  | 75   | 801             | 8,107   | 90   | 164             | 589     | 105  | 1               | 3       |
| 61   | 942             | 20,872  | 76   | 768             | 7,306   | 91   | 130             | 425     | 106  | ·25             | 2       |
| 62   | 943             | 19,930  | 77   | 733             | 6,538   | 92   | 87              | 295     | 107  | ·25             | 1·75    |
| 63   | 944             | 18,987  | 78   | 697             | 5,805   | 93   | 60              | 208     | 108  | ·25             | 1·50    |
| 64   | 943             | 18,043  | 79   | 654             | 5,108   | 94   | 44              | 148     | 109  | ·25             | 1·25    |
| 65   | 942             | 17,100  | 80   | 610             | 4,454   | 95   | 31              | 104     | 110  | ·25             | 1·0     |
| 66   | 939             | 16,158  | 81   | 559             | 3,844   | 96   | 19              | 73      | 111  | ·25             | ·75     |
| 67   | 933             | 15,219  | 82   | 513             | 3,285   | 97   | 14              | 54      | 112  | ·25             | ·50     |
| 68   | 926             | 14,286  | 83   | 460             | 2,772   | 98   | 9               | 40      | 113  | ·25             | ·25     |
| 69   | 915             | 13,360  | 84   | 408             | 2,312   | 99   | 6               | 31      | 114  | 0               | 0       |
| 70   | 903             | 12,445  | 85   | 357             | 1,904   | 100  | 6               | 25      |      |                 |         |
| 71   | 888             | 11,542  | 86   | 307             | 1,547   | 101  | 5               | 19      |      |                 |         |
| 72   | 871             | 10,654  | 87   | 258             | 1,240   | 102  | 5               | 14      |      |                 |         |
| 73   | 850             | 9,783   | 88   | 215             | 982     | 103  | 4               | 9       |      |                 |         |
| 74   | 826             | 8,933   | 89   | 178             | 767     | 104  | 2               | 5       |      |                 |         |

I shall take this opportunity of endeavouring to demonstrate, in a simple and undeniable manner, the error into which Dr. Price and his followers have fallen, in consequence, as it appears, of their adopting the legal restraints on usury as essential steps in the mathematical calculation of the amount of compound interest. The error has, indeed, of late years been very commonly admitted; but its effects have not been so satisfactorily rectified as could be desired.

In the 66th volume of the *Philosophical Transactions*, for the year 1776, we find a paper of Dr. Price, in which he lays down these theorems,  $r$  denoting the interest of £1 for a year, and  $n$  the term or number of years during which any annuity will be paid;  $p$  the perpetuity, or  $\frac{1}{r}$ ;  $y$  the value of an annuity paid yearly, and  $h$  half yearly. Then, I.,  $y = p - \frac{1}{r(1+r)^n}$ ; and, II.,  $h = p - \frac{1}{r\left(1 + \frac{r}{2}\right)^{2n}}$ : and as examples, taking  $r = \cdot 04$ , and  $n = 5$ , we have  $y = 4\cdot 4518$ , and  $p = 4\cdot 4913$ .

Now, if we analyze the results thus obtained, by dividing them into the present values of the separate payments, they will stand thus:—

|    |  |           |
|----|--|-----------|
| I. | Present value of £1 payable at the end of 1 year | £·961538  |
|    | 2 years  | ·924556   |
|    | 3 "  | ·888996   |
|    | 4 "  | ·854804   |
|    | 5 "  | ·821927   |
|    |  | <hr/>     |
|    |  | £4·451821 |

|  |   |   |                  | £.             |
|--|---|---|------------------|----------------|
| II. Present value of 10s., payable at the end of half a year |   |   |                  | ·49020         |
| "  | " | " | 1 year           | ·48058         |
| "  | " | " | $1\frac{1}{2}$ " | ·47127         |
| "  | " | " | 2 "              | ·46192         |
| "  | " | " | $2\frac{1}{2}$ " | ·45286         |
| "  | " | " | 3 "              | ·44398         |
| "  | " | " | $3\frac{1}{2}$ " | ·43528         |
| "  | " | " | 4 "              | ·42674         |
| "  | " | " | $4\frac{1}{2}$ " | ·41837         |
| "  | " | " | 5 "              | ·41018         |
|  |   |   |                  | <hr/> £4·49138 |

The present values of 10s. are therefore assumed :

|              |        |     |        |
|--------------|--------|-----|--------|
| I. At 1 year | ·48077 | II. | ·48058 |
| 2 years      | ·46228 |     | ·46192 |
| 3 "          | ·44450 |     | ·44398 |
| 4 "          | ·42740 |     | ·42674 |
| 5 "          | ·41096 |     | ·41018 |

The latter column exhibiting obviously a larger deduction for discount than the former, so that the rate of interest in the two calculations is by no means the same; although, in the case of  $r = \cdot 05$ , they would respectively represent the highest rate of interest allowed by our laws to be received without a new investment or engagement: but this arbitrary restraint ought certainly not to affect the mathematical consideration of the question.

The difficulty, if any person thinks it such, may be avoided by a mode of investigation which I have lately had occasion to point out:—"An annuity, of which a payment is due on a given day, is more valuable than an annuity purchased on that day, and to commence a year after, by the amount of a year's payment: and *the value of a life annuity becoming payable at any intermediate time between the day of purchase and its first anniversary will be greater than the simple tabular value of the annuity by a sum proportional to the anticipation of the payment;*" the increase of the value being very nearly uniform, when we suppose the anticipation to be gradually increased: this increase of the value comprehending obviously the greater probability as well as the greater proximity of each payment, and proceeding from day to day by very nearly equal increments. Thus, if we wished to purchase an annuity of £100 a year, and its value were £1,000, upon the ordinary supposition of the payments commencing after the end of a year—supposing that we desired to have the first payment made at the end of nine months, and the subsequent payments at annual intervals as usual—we should have to add £25 to the purchase-

money, making it £1,025, at whatever rate of interest the value might have been computed. If we began at six months, £50, and if at three months, £75, must be added to the purchase: it being obvious that an additional £100 would be equivalent to an anticipation of twelve months, or to an immediate payment of a year's annuity.

From this simple and incontestable principle, in which the second differences only are neglected, it is very easy to deduce the values of annuities payable at intervals shorter than a year. An annuity of 1, payable half yearly, is equal to two annuities of  $\frac{1}{2}$ , the one beginning as usual at the end of the year, the other anticipated by half a year; and the value of this portion is greater than the other by half of one of the payments, that is, by  $\frac{1}{4}$ : so that "*we may always find the value of a life annuity payable half yearly, by adding a quarter of a year to the tabular value of the same annuity.*"

In a similar manner it is very easily shown, that "*for quarterly payments we must add  $\frac{3}{8}$  of a year's value to the computation made on the supposition of annual payments;*" and "*the continual bisection of the interval would at last afford us the addition of half a yearly payment for the value of a daily or hourly payment of a proportional part of the given annuity.*"

"It may also be observed, that when we reckon at 3 per cent. interest, an annuity payable half yearly is the same, throughout the middle of life, that would be granted on the life of a person a year older, if payable annually."

If it is required to ascertain the value of a reversionary annuity payable half yearly or quarterly, the calculation becomes in appearance a little paradoxical; for since the true value of a reversionary annuity for the life of one person, for example, after the death of another, is the difference between the values of two annuities on the single life and the joint lives, and since an equal addition must be made to these values in consideration of the period of payment being shortened, it follows that the reversionary annuity must be of equal value in either form. This conclusion would indeed be strictly true if the periodical times of payment remained unaltered, according to the supposition from which the value of the annuities is deduced; while, in fact, it is usual to grant such an annuity to commence at the first quarterly, half yearly, or annual period after the contingent event—a variation which would have no sensible effect in the case of daily payments, but which lessens the value of reversionary annuities at other periods by that of half a pay-

ment for the given period, reduced to the present time in the same manner as any other sum assured as payable upon the same contingency of survivorship.

The simplicity observable in the progression of the values of annuities, calculated according to the values of lives here supposed, and at 3 per cent. interest, leads us to inquire what would be the exact law of mortality required to make that progression strictly uniform throughout life; and it will appear on investigation, that in order to have the value  $24.45 - \frac{1}{4}x$ ,  $x$  being the age of the person, which is nearly true between 20 and 70, the annual mortality must be expressed by  $\frac{.03x + .066}{100.8 - x}$ : a fraction which at 20 becomes  $\frac{1}{121}$ ; at 40,  $\frac{1}{48}$ ; at 60,  $\frac{1}{22}$ ; and at 80,  $\frac{1}{8.4}$ . Our table gives respectively  $\frac{1}{103}$ ,  $\frac{1}{50}$ ,  $\frac{1}{23}$ , and  $\frac{1}{7.3}$ ; the Northampton,  $\frac{1}{71}$ ,  $\frac{1}{48}$ ,  $\frac{1}{25}$ , and  $\frac{1}{7.4}$ . Mr. Finlaison's male annuitants,  $\frac{1}{87}$ ,  $\frac{1}{73}$ ,  $\frac{1}{32}$ , and  $\frac{1}{8.3}$ . The healthiness of Mr. Finlaison's annuitants about 40 and 50 is one of the most remarkable features of his table. He observes (p. 58), that out of 10,000 persons at 23, 141 will die in a year, and 141 will die out of the same number at the age of 48; but at the age of 34 there will only die 124. The curve marked by obelisks (+) in the diagram will show the comparative progress of mortality in this system; which, however valuable the data may be, appears to exhibit too many novelties, if not anomalies, to be generally adopted with confidence: while the line of crosses (x), representing the Tontine of Deparcieux, will serve to show how little difference the lapse of a century has made in the results of these two similar cases.

I shall conclude with a comparison of the climacteric years, as they may be called without impropriety, in which the greatest numbers of adults die, as taken from different tables.

I sincerely hope that these considerations may help to undeceive the too credulous public, who have of late not only received some hints that tend to insinuate the probability of an occasional recurrence of a patriarchal longevity, but who have been required to believe, upon the authority of a most respectable mathematician, that the true and unerring value of life is not to be obtained by taking an average of various decrements, but by adopting the extreme of all conceivable estimates, founded only on a hasty assertion of Mr. Morgan, and unsupported by any detailed report:

an estimate which makes the grand climacteric of mankind in this country, not a paltry fifty-four, or the too much dreaded sixty-three; but no less than EIGHTY-TWO! an age to which nearly one sixth of the survivors at ten are supposed to attain!

*Climacterics, or greatest Decrements.*

|                              |    |                             |    |                           |    |
|------------------------------|----|-----------------------------|----|---------------------------|----|
| Berlin, formerly . . . . .   | 38 | Formula . . . . .           | 63 | Sweden, 1785 . . . . .    | 69 |
| London, about 1733 . . . . . | 40 | Brandenburg . . . . .       | 65 | Holycross, 1760 . . . . . | 70 |
| Paris, formerly . . . . .    | 40 | Warrington, 1777 . . . . .  | 65 | Deparcieux . . . . .      | 73 |
| Stockholm, 1762 . . . . .    | 42 | Norwich, 1765 . . . . .     | 66 | Carlisle . . . . .        | 74 |
| London, 1764 . . . . .       | 43 | Montpellier, 1782 . . . . . | 67 | Ackworth, 1752 . . . . .  | 75 |
| London, 1815 . . . . .       | 46 | Duvillard, France . . . . . | 67 | Kerseboom . . . . .       | 77 |
| Northampton, 1757 . . . . .  | 56 | Sweden, 1762 . . . . .      | 68 | Finlaison . . . . .       | 78 |
| Breslau, 1695 . . . . .      | 61 | Chester, 1776 . . . . .     | 68 | E. O. Mr. B. . . . .      | 82 |

NOTE.—Some of the tables appended to this paper have been omitted, as devoid of interest at the present day—ED. A. M.

*On the Settlement of Losses by Fire under Average Policies. By*  
 RICHARD ATKINS, Esq., of the Sun Fire Office.

IT was intended that the short essay descriptive of the existing system of settlements of claims under average fire policies, which appeared in this *Journal* as far back as No. X. (January, 1853), should have been promptly followed by some remarks on the most obvious defects of the system, with a few practical suggestions for their remedy. It is not necessary to explain here the reasons for the delay which has taken place in the fulfilment of that intention; but it is satisfactory to know that the postponement has not been without its use. Many valuable hints have been offered; and although some marked differences of opinion exist as to any proposed remedies, the explanations given of the existing rules are, without any exception, admitted to be correct.

The examples of some remarkably involved claims and their ultimate settlement, which were given in the former article, have been thought serviceable; and suggestions have been made that a considerable number and variety of similar cases of actual settlements should be collected and published, in order that these important precedents may be generally known. It does not, of course, follow, that any previous decisions should come with such a weight of authority as to preclude a fresh discussion on many of these doubtful points; but in the absence of any fixed or acknowledged law it is unquestionably of some value to know what judgments